Let ϕ be the Cantor-Lebesgue function and define

$$\psi(x) = \frac{1}{2}(\phi(x) + x)$$

Then ψ is a strictly increasing function from [0,1] onto [0,1], and maps a measurable subset of Cantor set onto an non-measurable set. (proposition on Royden's real analysis)

So we can suppose E is a subset of Cantor set and $\psi(E)$ is non-measurable.

Define $f : [-1, 1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} \psi(x) & x \in E \\ x^2 + 2 & x \in [-1, 1] \setminus E \end{cases}$$

$f(x)$ is continuous outside E, which is of measure zero. So $f(x)$ is measurable.

If $n(y) := \#\{x : f(x) = y\}$, then

$$\{y : n(y) = 1\} = \psi(E) \cup \{x^2 + 2 : x \in E\}$$

which is not a measurable set since $\psi(E)$ is not measurable and $\{x^2 + 2 : x \in E\}$ is far away from $\psi(E)$.

Hence $n(y)$ is not measurable.