1. Suppose that $f$ is an integrable function on $\mathbb{R}^d$. For each $\alpha > 0$, let $E_\alpha = \{ x | |f(x)| > \alpha \}$. Prove that:

$$\int_{\mathbb{R}^d} |f(x)| dx = \int_0^\infty m(E_\alpha) d\alpha.$$  

2. Let $p(z)$ be a polynomial of degree $d \geq 2$, with distinct roots $a_1, a_2, \cdots, a_d$. Show that

$$\sum_{i=1}^d \frac{1}{p'(a_i)} = 0.$$  

3. Let $\alpha$ be a number such that $\alpha/\pi$ is not a rational number. Show that:

1) $\lim_{N \to \infty} \sum_{n=1}^N e^{ik(x+n\alpha)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{ikt} dt.$

2) For every continuous periodic function $f : \mathbb{R} \to \mathbb{C}$ of period $2\pi$, we have

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^N f(x + n\alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt.$$  

4. Let $u$ be a positive harmonic function over the punctured complex plane $\mathbb{C}/\{0\}$. Show that $u$ must be a constant function.

5. Suppose $H = L^2(B)$, $B$ is the unit ball in $\mathbb{R}^d$. Let $K(x,y)$ be a measurable function on $B \times B$ that satisfies

$$|K(x,y)| \leq A |x-y|^{-d+\alpha}$$

for some $\alpha > 0$, whenever $x, y \in B$. Define

$$Tf(x) = \int_B K(x,y)f(y)dy$$

(a) Prove that $T$ is a bounded operator on $H$.
(b) Prove that $T$ is compact.

6. Let $A$ be an $n \times n$ real non-degenerate symmetric matrix. For $\lambda \in \mathbb{R}^+$, we define: $\int_{\mathbb{R}} \exp(i\lambda x^2) dx = \lim_{\epsilon \to 0^+} \int_{-\infty}^{-\epsilon} \exp(i\lambda x^2 - \frac{1}{2}\epsilon x^2) dx$. Show that:
\[
\int_{\mathbb{R}^n} \exp(\lambda \frac{i}{2} < Ax, x > - i < x, \xi >) dx \\
= (\frac{2\pi}{\lambda})^{n/2} |\text{det}(A)|^{-1/2} \exp(- \frac{i}{2\lambda} < A^{-1} \xi, \xi >) \exp(\frac{i\pi}{4} \text{sgn}A).
\]

Here \( \lambda \in \mathbb{R}^+, \xi \in \mathbb{R}^n, \text{sgn}(A) = \nu_+(A) - \nu_-(A), \nu_+(A)(\nu_-(A)) \) is the number of positive (negative) eigenvalues of \( A \).