The 8^{th} Romanian Master of Mathematics Competition

Day 1: Friday, February 26, 2016, Bucharest

Language: English

Problem 1. Let ABC be a triangle and let D be a point on the segment BC, $D \neq B$ and $D \neq C$. The circle ABD meets the segment AC again at an interior point E. The circle ACD meets the segment AB again at an interior point F. Let A' be the reflection of A in the line BC. The lines $A'C$ and DE meet at P, and the lines $A'B$ and DF meet at Q. Prove that the lines AD, BP and CQ are concurrent (or all parallel).

Problem 2. Given positive integers m and $n \geq m$, determine the largest number of dominoes (1×2 or 2×1 rectangles) that can be placed on a rectangular board with m rows and $2n$ columns consisting of cells (1×1 squares) so that:

(i) each domino covers exactly two adjacent cells of the board;
(ii) no two dominoes overlap;
(iii) no two form a 2×2 square; and
(iv) the bottom row of the board is completely covered by n dominoes.

Problem 3. A *cubic sequence* is a sequence of integers given by $a_n = n^3 + bn^2 + cn + d$, where b, c and d are integer constants and n ranges over all integers, including negative integers.

(a) Show that there exists a cubic sequence such that the only terms of the sequence which are squares of integers are a_{2015} and a_{2016}.

(b) Determine the possible values of $a_{2015} \cdot a_{2016}$ for a cubic sequence satisfying the condition in part **(a)**.

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.