Problem 4. Let $x$ and $y$ be positive real numbers such that $x + y^{2016} \geq 1$. Prove that $x^{2016} + y > 1 - 1/100$.

Problem 5. A convex hexagon $A_1B_1A_2B_2A_3B_3$ is inscribed in a circle $\Omega$ of radius $R$. The diagonals $A_1B_2$, $A_2B_3$, and $A_3B_1$ concur at $X$. For $i = 1, 2, 3$, let $\omega_i$ be the circle tangent to the segments $XA_i$ and $XB_i$, and to the arc $A_iB_i$ of $\Omega$ not containing other vertices of the hexagon; let $r_i$ be the radius of $\omega_i$.

(a) Prove that $R \geq r_1 + r_2 + r_3$.

(b) If $R = r_1 + r_2 + r_3$, prove that the six points where the circles $\omega_i$ touch the diagonals $A_1B_2$, $A_2B_3$, $A_3B_1$ are concyclic.

Problem 6. A set of $n$ points in Euclidean 3-dimensional space, no four of which are coplanar, is partitioned into two subsets $A$ and $B$. An $AB$-tree is a configuration of $n - 1$ segments, each of which has an endpoint in $A$ and the other in $B$, and such that no segments form a closed polyline. An $AB$-tree is transformed into another as follows: choose three distinct segments $A_1B_1$, $B_1A_2$, and $A_2B_2$ in the $AB$-tree such that $A_1$ is in $A$ and $A_1B_1 + A_2B_2 > A_1B_2 + A_2B_1$, and remove the segment $A_1B_1$ to replace it by the segment $A_1B_2$. Given any $AB$-tree, prove that every sequence of successive transformations comes to an end (no further transformation is possible) after finitely many steps.

Each of the three problems is worth 7 points.
Time allowed $4\frac{1}{2}$ hours.